University of Mumbai

PROPOSED SYLLABUS OF

S.Y.B.A/B.Sc. (Mathematics)

(Effective from June 2009)

The Preamble

Mathematics, on the one hand, has purity and beauty, and on the other, is a contemporary subject whose concepts and methodology are being used by working Physicists, Statisticians, Computer Scientists, Chemists and Biologists. Therefore, in the new S.Y.B.A./B.Sc syllabus, the basic and fundamental topics of mathematics are included along with the applicable ones.

There are three papers of Mathematics for Science students and the first two are for Arts students.

Paper I is **Calculus and Analysis**. The fundamental concepts and techniques of real analysis have been introduced and there is a gentle transition to rigourous mathematics. The paper also includes differential equations, multiple integrals and integration of vector fields. The topic of differential equations is the most important part of mathematics for understanding the physical sciences. It is also the source of many ideas and theories which constitute Higher Analysis. The multiple integrals are generalizations of definite integrals of functions of one variable. The line integrals arise naturally in Mechanics as work done by a force.

Paper II is **Linear Algebra**. Linear Algebra has rigour and purity and is as applicable as Calculus. The subject was taught in an abstract manner a few years ago. In the abstract treatment, the crucial importance of the subject is missed. In this syllabus, visualization and geometric interpretations are emphasized. Linear Algebra has applications in networks and game theory.

Paper III (only for Science students) is **Computational Mathematics**. This paper includes the topics of algorithms, graphs, trees, (Discrete Mathematics) and applications of integration and numerical methods. Algorithms are not just useful in Computer Science; constructing efficient algorithms is an important way of solving mathematical problems too, as seen with the Euclidean algorithm. The topic of the complexity of algorithms is introduced. Graphs and trees are useful methods for solving many problems. Important applications of integration such as measuring areas, volumes, improper integrals and gamma function are included. Very often, the solutions of mathematical problems cannot be obtained by an analytic method but can be described in an approximate manner by numerical methods. The numerical methods of finding approximate roots of equations, LU factorization of matrices, numerical methods to find solutions of differential equations have also been included.

Tutorials/assignments/project - case studies have been prescribed in Paper I and Paper II. It is important to develop among the students an ability to understand and apply the theoretical concepts. The teachers can attempt to accomplish this task during tutorials. Thought provoking exercises along with routine ones may be given to students for assignments.

A student may write a project/case study instead of an assignment in Paper I and/or Paper II. It helps the student to arrive at a deeper understanding and wider significance of the concept. The concepts of Mathematics are re-inforced by genuine applications.

Paper III contains a component of practicals. It is advisable to form groups in a batch and the same practical experiment be conducted using different methods if applicable to get the feel of the solution space for solving problems.

This syllabus should motivate the students and make them appreciate the beauty and value of Mathematics.

S.Y.B.Sc & S.Y.B.A

Teaching Pattern

		Title
S.Y.B.Sc.	Paper I	Calculus and Analysis
	Paper II	Linear Algebra
	Paper III	Computational Mathematics
S.Y.B.A.	Paper I	Calculus and Analysis
	Paper II	Linear Algebra

Teaching Pattern

1. Three lectures per week per paper.
2. One tutorial per week per batch per paper for Paper I and Paper II. (The batches of tutorials to be formed as prescribed by the University).
3. One assignment per unit **OR** a Project/Case Study in either of Paper I, Paper II or both. The project work can be guided by a faculty member of the college/Institute/Mumbai University/any other University.
4. One practical per week per batch for S.Y.B.Sc. Paper III. (The batches are to be formed as prescribed by the University).
5. Teaching aids such as Computer Algebra Systems may be used.

S.Y.B.A/B.Sc. Paper I

Calculus and Analysis

Term I

Unit 1. Real Numbers (15 Lectures)

(a) (i) Statements of algebraic and order properties of $\mathbb{R}$.

(ii) Elementary consequences of these properties including the A.M. - G.M. inequality, Cauchy-Schwarz inequality, and Bernoulli inequality (without proof).

(b) (i) Review of absolute value and neighbourhood of a real number.

(ii) Hausdorff property.

(c) Supremum (lub) and infimum (glb) of a subset of $\mathbb{R}$, lub axiom of $\mathbb{R}$, Consequences of lub axiom of $\mathbb{R}$ including

(i) Archimedian property.

(ii) Density of rational numbers.

(iii) Existence of n^{th} root of a positive real number (in particular square root).

(iv) Decimal representation of a real number.

(d) (i) Nested Interval Theorem.

(ii) Open sets in $\mathbb{R}$ and closed sets as complements of open sets.

(iii) Limit points of a susbset of $\mathbb{R}$, examples, characterisation of a closed set as a set containing all its limit points.

(e) Open cover of a susbset of $\mathbb{R}$, Compact susbset of $\mathbb{R}$, Definition and examples. A closed and bounded interval $[a, b]$ is compact.

Reference for Unit 1: *Chapter II*, *Sections* 1, 2, 4, 5, 6 and *Chapter X*, *Sections* 1, 2 *of* **Introduction to Real Analysis**, Robert G. Bartle and Donald R. Sherbet, *Springer Verlag.*

Unit 2. Sequences, Limits and Continuity (15 Lectures)

(a) Sequence of real numbers, Definition and examples. Sum, difference, product, quotient and scalar multiple of sequences.

(b) Limit of a sequence, Convergent and divergent sequences, Uniqueness of limit of a convergent sequence, Algebra of convergent sequences, Sandwich Theorem of sequences. Limits of standard sequences such as

$$\left\{\frac{1}{n^\alpha}\right\}\ \alpha > 0,\quad \{a^n\}\ |a| < 1,\ \{n^{1/n}\},\ \{a^{1/n}\}\ a > 0,\ \left\{\frac{1}{n!}\right\},\ \left\{\frac{a^n}{n!}\right\}\ a \in \mathbb{R}$$

Examples of divergent sequences.

(c) (i) Bounded sequences, A convergent sequence is bounded.

(ii) Monotone sequences, Convergence of bounded monotone sequences, The number e as a limit of a sequence, Calculation of square root of a positive real number.

(d) (i) Subsequences.

(ii) Limit inferior and limit superior of a sequence.

(iii) Bolzano-Weierstrass Theorem of sequences.

(iv) Sequential characterisation of limit points of a set.

(e) Cauchy sequences, Cauchy completeness of $\mathbb{R}$.

(f) Limit of a real valued function at a point

(i) Review of the $\varepsilon - \delta$ definition of limit of functions at a point, uniqueness of limits of a function at a point whenever it exists.

(ii) Sequential characterization for limits of functions at a point, Theorems of limits (Limits of sum, difference, product, quotient, scalar multiple and sandwich theorem).

(iii) Continuity of function at a point, $\varepsilon{-}\delta$ definition, sequential criterion, Theorems about continuity of sum, difference, product, quotient and scalar multiple of functions at a point in the domain using $\varepsilon - \delta$ deinition **or** sequential criterion. Continuity of composite functions. Examples of limits and continuity of a function at a point using sequential criterion.

(iv) A continuous function on closed and bounded interval is bounded and attains bounds.

Reference for Unit 2: *Chapter III*, *Sections* 1, 2, 3, 4, 5, *Chapter IV*, *Sections* 1, 2 and *Chapter V*, *Sections* 1, 2, 3 *of* **Introduction to Real Analysis**, Robert G. Bartle and Donald R. Sherbet, *Springer Verlag*.

Unit 3. Infinite Series (15 Lectures)

(a) Infinite series of real numbers, The sequence of partial terms of an infinite series, convergence and divergence of series, sum, difference and multiple of convergent series are again convergent.

(b) Cauchy criterion of convergence of series. Absolute convergence of a series, Geometric series.

(c) Alternating series, Leibnitz' Theorem, Conditional convergence, An absolutely convergent series is conditionally convergent, but the converse is not true.

(d) Rearrangement of series (without proof), Cauchy condensation test (statement only), application to convergence of p - series $\sum \frac{1}{n^p}$ $(p > 1)$. Divergence of Harmonic series $\sum \frac{1}{n}$.

(e) Tests for absolute convergence, Comparison test, Ratio test, Root test.

(f) Power series, Radius of convergence of power series, The exponential, sine and cosine series.

(g) Fourier series, Computing Fourier Coefficients of simple functions such as x, x^2, $|x|$, piecewise continuous functions on $[-\pi, \pi]$.

Reference for Unit 3: *Chapter IX*, *Sections* 1, 2, 3, 4 and *Chapter VIII*, *Sections* 3, 4 *of* **Introduction to Real Analysis**, Robert G. Bartle and Donald R. Sherbet, *Springer Verlag.*

Term II

Unit 4. Differential Equations (15 Lectures)

(a) First Order Differential Equations:

  (i) Review of separable differential equations, homogeneous and non-homogeneous differential equations.

  (ii) Exact differential equations and integrating factors.
  Rules for finding integrating factors of $M(x, y)dx + N(x, y)dy = 0$ (without proof) when,
  - $\dfrac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = h(y)$,
  - $\dfrac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = g(x)$,
  - $\dfrac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{yN - xM} = f(xy)$.

  (iii) Linear differential equations and Bernoulli differential equations.

  (iv) Modeling with first order equations. Examples from Financial Mathematics, Chemistry, Environmental Science, Population growth and decay.

(b) Second order Linear Differential Equations:

  (i) The general second order linear differential equation. Existence and Uniqueness Theorem for the solutions of a second order initial value problem (statement only).

(ii) Homogeneous and non-homogeneous second order linear differential equations:
  - The space of solutions of the homogeneous equations as a vector space.
  - Wronskian and linear independence of the solutions.
  - The general solution of homogeneous differential equation. The use of known solutions to find the general solution of a homogeneous equations.
  - The general solution of a non-homogeneous second order equation, Complementary functions and particular integrals.

(iii) The homogeneous equation with constant coefficients, auxiliary equation, the general solution corresponding to real and distinct roots, real and equal roots and complex roots of the auxiliary equation.

(iv) Non-homogeneous equations: The method of undetermined coefficients. The method of variation of parameters.

Reference for Unit 4: *Chapter* 2, *Sections* 7, 8, 9, 10 and *Chapter* 3, *Sections* 14, 15, 16, 17, 18, 19, 20 *of* **Differential Equations with Applications and Historical Notes**, G.F. SIMMONS, *McGraw Hill* and *Chapter* 1, *Sections* 1, 2, 3 *of* **Elements of Partial Differential Equations**, I. SNEDDON *McGraw Hill*.

Unit 5. Multiple integrals (15 Lectures)

Review of functions of two and three variables, partial derivatives and gradient of two or three variables.

(a) Double integrals:

  (i) Definition of double integrals over rectangles.

  (ii) Properties of double integrals.

  (iii) Double integrals over bounded regions.

(b) Statement of Fubini's Theorem, Double integrals as volumes.

(c) Applications of Double integrals: Average value, Areas, Moments, Center of Mass.

(d) Double integrals in polar form.

(e) Triple integrals in Rectangular coordinates, Average, volumes.

(f) Applications of Triple integrals: Mass, Moments, Parallel axis Theorem.

(g) Triple integrals in Spherical and Cylindrical coordinates.

Reference for Unit 5: *Chapter* 13, *Sections* 13.1, 13.2, 13.3, 13.4, 13.5, 13.6 *of* **Calculus and Analytic Geometry**, G.B. THOMAS *and* R. L. FINNEY, *Ninth Edition*, *Addison-Wesley*, 1998.

Unit 6. Integration of Vector Fields (15 Lectures)

(a) Line Integrals, Definition, Evaluation for smooth curves.

Mass and moments for coils, springs, thin rods.

(b) Vector fields, Gradient fields, Work done by a force over a curve in space, Evaluation of work integrals.

(c) Flow integrals and circulation around a curve.

(d) Flux across a plane curve.

(e) Path independence of the integral $\int_A^B F \cdot dr$ in an open region, F being a vector field over the region and A, B points in the region.

Conservative fields, potential function.

(f) The Fundamental theorems of line integrals (without proof).

(g) Flux density (divergence), Circulation density (curl) at a point.

(h) Green's Theorem in plane (without proof), Evaluation of line integrals using Green's Theorem.

Reference for Unit 6: *Chapter* 14 *of* 14.1, 14.2, 14.3, 14.4 **Calculus and Analytic Geometry**, G.B. THOMAS *and* R. L. FINNEY, *Ninth Edition*, *Addison-Wesley*, 1998.

The proofs of the results mentioned in the syllabus to be covered unless indicated otherwise.

Recommended Books

1. ROBERT G. BARTLE and DONALD R. SHERBET : **Introduction to Real Analysis**, *Springer Verlag.*

2. R. COURANT and F. JOHN : **Introduction to Calculus and Analysis Vol I**, Reprint of First Edition, *Springer Verlag*, New York 1999.

3. R. R. GOLDBERG: **Methods of Real Analysis**, *Oxford and IBH Publication Company*, New Delhi.

4. T. APOSTOL: **Calculus Vol I**, Second Edition, *John Wiley.*

5. M. H. PROTTER: **Basic elements of Real Analysis**, *Springer Verlag*, New York 1998.

6. G.B. THOMAS and R. L. FINNEY, **Calculus and Analytic Geometry**, Ninth Edition, Addison-Wesley, 1998.

7. G.F. Simmons: **Differential Equations with Applications and Historical Notes**, *McGraw Hill.*

8. I. Sneddon: **Elements of Partial Differential Equations**, *McGraw Hill.*

Additional Reference Books

1. Howard Anton, *Calculus - A new Horizon*, Sixth Edition, John Wiley and Sons Inc, 1999.
2. James Stewart, *Calculus*, Third Edition, Brooks/cole Publishing Company, 1994.
3. E.A. Coddington and R. Carlson : *Linear Ordinary Differential Equations*, SIAM.
4. W.E. Boyce and R.C. Diprima: *Elementary Differential equations and Boundary value problems* , John Wiley and Sons 8th Edition.
5. A.H. Siddiqi and P. Manchanda : *A First Course in Differential Equations with Applications*, Macmillan.

Suggested topics for Tutorials/Assignments

(1) Properties of real numbers and Hausdorff property.
(2) Bounded sets, finding l.u.b. and g.l.b. of sets.
(3) Archmedian Property and Density Theorem.
(4) Nested Interval and decimal representations.
(5) Finding limit points of given sets.
(6) Compact sets.
(7) (i) Find limits of sequences using definition.
 (ii) Monotone sequences.
(8) Subsequences, finding limit inferior and limit superior of given sequences.
(9) Cauchy sequences.
(10) Limits and continuity using sequential criterion.
(11) Convergence of series. Comparison test.
(12) Convergence of series: Root test, Ratio test.
(13) Radius of convergence of a power series.
(14) Fourier Series.
(15) Solving first order exact equations and non-exact equations using integrating factors.
(16) Linear equations, Bernoulli equations, Euler's equations
(17) Wronskian and linear independence of solutions
(18) Second order linear homogeneous equatins with constant coefficient.
(19) Method of undetermined coefficients, Method of variation of parameters
(20) Double integrals, sketching regions, evaluation.
(21) Triple integrals.

(22) Application of Double and Triple integrals.

(23) Evaluation of line integrals using definition, calculation of mass and moments for coil etc.

(24) (i) Evaluation of work integrals

(ii) Flow integrals and flux across a plane curve.

(iii) Conservative fields and potential functions.

S.Y.B.A/B.Sc. Paper II

Linear Algebra

Term I

Unit 1. Systems of linear equations and matrices (15 Lectures)

(a) Systems of homogeneous and non-homogeneous linear equations.

  (i) The solutions of systems of m homogeneous linear equations in n unknowns by elimination and their geometric interpretation for $(m, n) = (1, 2)$, $(1, 3)$, $(2, 2)$, $(2, 3)$, $(3, 3)$.

  (ii) The existence of non-trivial solution of such a system for $m < n$. The sum of two solutions and a scalar multiple of a solution of such a system is again a solution of the system.

(b) (i) Matrices over $\mathbb{R}$, The matrix representation of systems of homogeneous and non-homogeneous linear equations.

  (ii) Addition, scalar multiplication and multiplication of matrices, Transpose of a matrix.

  (iii) The types of matrices: zero matrix, identity matrix, symmetric and skew symmetric matrices, upper and lower triangular matrix.

  (iv) Transpose of product of matrices, Invertible matrices, Product of invertible matrices.

(c) (i) Elementary row operations on matrices, row echelon form of a matrix and Gaussian elimination method. Applications of Gauss elimination method to solve system of linear equations.

  (ii) The matrix units, Row operations and Elementary matrices, Elementary matrices are invertible and an invertible matrix is a product of elementary matrices.

Reference for Unit 1: *Chapter II*, *Sections* 1, 2, 3, 4, 5 *of* **Introduction to Linear Algebra**, Serge Lang, *Springer Verlag* and *Chapter 1*, *of* **Linear Algebra A Geometric Approach**, S. Kumaresan, *Prentice-Hall of India Private Limited*, New Delhi.

Unit 2. Vector spaces over $\mathbb{R}$ (15 Lectures)

(a) Definition of a vector space over $\mathbb{R}$. Examples such as:

  (i) Euclidean space $\mathbb{R}^n$.

  (ii) The space $\mathbb{R}^\infty$ of sequences over $\mathbb{R}$.

  (iii) The space of $m \times n$ matrices over $\mathbb{R}$.

  (iv) The space of polynomials with real coefficients.

(v) The space of real valued functions on a non-empty set.

(b) Subspaces - definition and examples including:

(i) Lines in $\mathbb{R}^2$, Lines and planes in $\mathbb{R}^3$.

(ii) The solutions of homogeneous system of linear equations, hyperplane.

(iii) The space of convergent real sequences.

(iv) The spaces of symmetric, skew symmetric, upper triangular, lower triangular, diagonal matrices.

(v) The space of polynomials with real coefficients of degree $\leq n$.

(vi) The space of continuous real valued functions on $[a, b]$.

(vii) The space of continuously differentiable real valued functions on $[a, b]$.

(c) (i) The sum and intersection of subspaces, direct sum of vector spaces.

(ii) Linear combination of vectors, convex sets, linear span of a subset of a vector space.

(iii) Linear dependence and independence of a set.

(d) (The discussion of concepts mentioned below for finitely generated vector spaces only) Basis of a vector space, basis as a maximal linearly independent set and a minimal set of generators. Dimension of a vector space.

(e) (i) Row space, Column space of an $m \times n$ matrix over $\mathbb{R}$ and row rank, column rank of a matrix

(ii) Equivalence of row rank and column rank, Computing rank of a matrix by row reduction.

Reference for Unit 2: *Chapter* III, *Sections* 1, 2, 3, 4, 5, 6 *of* **Introduction to Linear Algebra**, Serge Lang, *Springer Verlag* and *Chapter 2*, *of* **Linear Algebra A Geometric Approach**, S. Kumaresan, *Prentice-Hall of India Private Limited*, New Delhi. .

Unit 3. Inner Product Spaces (15 Lectures)

(a) Dot product in $\mathbb{R}^n$, Definition of general inner product on a vector space over $\mathbb{R}$.

Examples of inner product including the inner product $\langle f, g \rangle = \int_{-\pi}^{\pi} f(t)g(t)dt$ on $C[-\pi, \pi]$, the space of continuous real valued functions on $[-\pi, \pi]$.

(b) (i) Norm of a vector in an inner product space.
Cauchy-Schwarz inequality, triangle inequality.

(ii) Orthogonality of vectors, Pythagorus theorem and geometric applications in $\mathbb{R}^2$, projections on a line, The projection being the closest approximation.

(iii) Orthogonal complements of a subspace, Orthogonal Complements in $\mathbb{R}^2$ and $\mathbb{R}^3$.

(iv) Orthogonal sets and orthonormal sets in an inner product space.
Orthogonal and orthonormal bases.
Gram-Schmidt orthogonalization process, simple examples in $\mathbb{R}^3$, $\mathbb{R}^4$.

Reference for Unit 3: *Chapter* VI, *Sections* 1, 2 *of* **Introduction to Linear Algebra**, Serge Lang, *Springer Verlag* and *Chapter 5*, *of* **Linear Algebra A Geometric Approach**, S. Kumaresan, *Prentice-Hall of India Private Limited*, New Delhi.

Term II

Unit 4. Linear Transformations (15 Lectures)

(a) Linear transformations - definition and properties, examples including:

- (i) Natural projection from $\mathbb{R}^n$ to $\mathbb{R}^m$ $(n \geq m)$
- (ii) The map $L_A : \mathbb{R}^n \to \mathbb{R}^m$ defined by $L_A(X) = AX$, where A is an $m \times n$ matrix over $\mathbb{R}$
- (iii) Rotations and reflections in $\mathbb{R}^2$, Streching and Shearing in $\mathbb{R}^2$.
- (iv) Orthogonal projections in $\mathbb{R}^n$.
- (v) Functionals.

The linear transformation being completely determined by its values on basis.

(b)
- (i) The sum and scalar multiple of linear transformations from U to V where U, V are finite dimensional vector spaces over $\mathbb{R}$ is again a linear transformation.
- (ii) The sapce $L(U, V)$ of linear transformations from U to V.
- (iii) The dual space V^* where V is finite dimensional real vector space.

(c)
- (i) Kernel and image of a linear transformation.
- (ii) Rank-Nullity Theorem.
- (iii) The linear isomorphisms, inverse of a linear isomorphism.
- (iv) Composite of linear transformations.

(d)
- (i) Representation of a linear transformation from U to V, where U and V are finite dimensional real vector spaces by matrices with respect to the given ordered bases of U and V. The relation between the matrices of linear transformation from U to V with respect to different bases of U and V.
- (ii) Matrix of sum of linear transformations and scalar multiple of a linear transformation.
- (iii) Matrices of composite linear transformation and inverse of a linear transformation.

(e) Equivalence of rank of an $m \times n$ matrix A and rank of the linear transformation $L_A : \mathbb{R}^n \to \mathbb{R}^m$ $(L_A(X) = AX)$. The dimension of solution space of the system of linear equations $AX = 0$ equals $n-$rank A.

(f) The solutions of non-homogeneous systems of linear equations represented by $AX = B$.

  (i) Existence of a solution when $\text{rank}(A) = \text{rank}(A, B)$.

  (ii) The general solution of the system is the sum of a particular solution of the system and the solution of the associated homogeneous system.

Reference for Unit 4: *Chapter* VIII, *Sections* 1, 2 *of* **Introduction to Linear Algebra**, Serge Lang, *Springer Verlag* and *Chapter 4*, *of* **Linear Algebra A Geometric Approach**, S. Kumaresan, *Prentice-Hall of India Private Limited*, New Delhi.

Unit 5. Determinants (15 Lectures)

(a) Definition of determinant as an n-linear skew-symmetric function from

$$\mathbb{R}^n \times \mathbb{R}^n \times \cdots \times \mathbb{R}^n \to \mathbb{R}^n$$

such that determinant of $(E^1, E^2, \ldots, E^n)$ is 1, where E^j denotes the j^{th} column of the $n \times n$ identity matrix I_n.

Determinant of a matrix as determinant of its column vectors (or row vectors).

(b) (i) Existence and uniqueness of determinant function via permutations.

  (ii) Computation of determinant of 2×2, 3×3 matrices, diagonal matrices.

  (iii) Basic results on determinants such as $\det(A^t) = \det(A)$, $\det(AB) = \det(A)\det(B)$.

  (iv) Laplace expansion of a determinant, Vandermonde determinant, determinant of upper triangular and lower triangular matrices.

(c) (i) Linear dependence and independence of vectors in $\mathbb{R}^n$ using determinants.

  (ii) The existence and uniqueness of the system $AX = B$, where A is an $n \times n$ matrix with $\det(A) \neq 0$.

  (iii) Cofactors and minors, Adjoint of an $n \times n$ matrix A.
  Basic results such as $A \cdot \text{adj}(A) = \det(A) \cdot I_n$. An $n \times n$ real matrix A is invertible if and only if $\det A \neq 0$; $A^{-1} = \dfrac{1}{\det A}(\text{adj}A)$ for an invertible matrix A.

  (iv) Cramer's rule.

(d) Determinant as area and volume.

Reference for Unit 5: *Chapter* VI *of* **Linear Algebra A geometric approach**, S. Kumaresan, *Prentice Hall of India Private Limited*, 2001 and *Chapter* VII **Introduction to Linear Algebra**, Serge Lang, *Springer Verlag*.

Unit 6. Eigenvalues and eigenvectors (15 Lectures)

(a) (i) Eigenvalues and eigenvectors of a linear transformation $T : V \to V$, where V is a finite dimensional real vector space.

(ii) Eigenvalues and eigenvectors of $n \times n$ real matrices and eigenspaces.

(iii) The linear independence of eigenvectors corresponding to distinct eigenvalues of a matrix (linear transformation).

(b) (i) The characteristic polynomial of an $n \times n$ real matrix, characteristic roots.

(ii) Similar matrices, characteristic polynomials of similar matrices.

(c) The characteristic polynomial of a linear transformation $T : V \to V$, where V is a finite dimensional real vector space.

Reference for Unit 6: *Chapter* VIII, *Sections* 1, 2 *of* **Introduction to Linear Algebra**, Serge Lang, *Springer Verlag* and *Chapter 7*, *of* **Linear Algebra A Geometric Approach**, S. Kumaresan, *Prentice-Hall of India Private Limited*, New Delhi.

The proofs of the results mentioned in the syllabus to be covered unless indicated otherwise.

Recommended Books

1. Serge Lang: *Introduction to Linear Algebra*, *Springer Verlag.*

2. S. Kumaresan: *Linear Algebra A geometric approach*, Prentice Hall of India Private Limited.

Additional Reference Books

1. M. Artin: *Algebra*, Prentice Hall of India Private Limited.

2. K. Hoffman and R. Kunze: *Linear Algebra*, Tata McGraw-Hill, New Delhi.

3. Gilbert Strang: *Linear Algebra and its applications*, International Student Edition.

4. L. Smith: *Linear Algebra*, Springer Verlag.

5. A. Ramachandra Rao and P. Bhima Sankaran: *Linear Algebra*, Tata McGraw-Hill, New Delhi.

6. T. Banchoff and J. Wermer: *Linear Algebra through Geometry*, Springer Verlag Newyork, 1984.

7. Sheldon Axler : *Linear Algebra done right*, Springer Verlag, Newyork.

8. Klaus Janich : *Linear Algebra.*

9. Otto Bretcher: *Linear Algebra with Applications*, Pearson Education.

10. Gareth Williams: *Linear Algebra with Applications*, Narosa Publication.

Suggested topics for Tutorials/Assignments

(1) Solving homogeneous system of m equations in n unknowns by elimination for $(m, n) = (1, 2)$, $(1, 3)$, $(2, 2)$, $(2, 3)$, $(3, 3)$.

(2) Row echelon form, Solving system $AX = B$ by Gauss elimination.

(3) Subspaces: Determining whether a given subset of a vector space is a subspace.

(4) Linear dependence and independence of subsets of a vector space.

(5) Finding bases of vector spaces.

(6) Rank of a matrix.

(7) Gram-Schmidt method.

(8) Orthogonal complements of subspaces of $\mathbb{R}^3$ (lines and planes).

(9) Linear transformations.

(10) Determining kernel and image of linear transformations.

(11) Matrices of linear transformations.

(12) Solutions of system of linear equations.

(13) Determinants: Computing determinants by Laplace's expansion.

(14) Applications of determinants: Cramer's rule.

(15) Finding inverses of 2×2, 3×3 invertible matrices using adjoint.

(16) Finding characteristic polynomial, eigenvalues and eigenvectors of 2×2 amd 3×3 matrices.

(17) Finding characteristic polynomial, eigenvalues and eigenvectors of linear transformations.

S.Y.B.Sc. Paper III

Computational Mathematics

Term I

Unit 1. Algorithms (15 Lectures)

(a) Definition of an algorithm, characteristics of an algorithm

Selection and iterative constructs in pseudocode, simple examples such as

(i) Finding the number of positive and negative integers in a given set,

(ii) Finding absolute value of a real number,

(iii) Exchanging values of variables,

(iv) Sum of n given numbers.

(b) Searching and sorting algorithms, including

(i) Finding maximum and/or minimum element in a finite sequence of integers.

(ii) The linear search and binary search algorithms of an integer x in a finite sequence of distinct integers.

(iii) Sorting of a finite sequence of integers in ascending order. Bubble sort and insertion sort.

(c) Algorithms on integers:

(i) Computing quotient and remainder in division algorithm.

(ii) Converting decimal number to a binary number.

(iii) Modular exponent.

(iv) Euclidean algorithm to find the g.c.d of two non-zero integers.

(d) Algorithms on matrices:

(i) Addition and multiplication of matrices.

(ii) Transpose of a matrix.

(iii) Power of a matrix.

(e) Complexity of algorithm: Big O notation, Growth of functions, Time complexity, Best case, Average case, Worst Case complexity.

Using big O notation to express the best, average and worst case behaviour for sorting and searching algorithms.

(f) Recursion, Examples including:

   (i) Fibonacci sequence
   (ii) Computing a^n for non-negative integer n.
   (iii) Euclidean algorithm.
   (iv) Searching algorithm
   (v) Factorial of a non-negative integer.

   Comparison of recursive and iterative methods.

Reference for Unit 1: *Chapter* 2, and *Chapter* 3, *Sections* 3.4, 3.5 *of* **Discrete Mathematics and Its Applications**, Kenneth H. Rosen, *McGraw Hill Edition.*

Unit 2. Graphs (15 Lectures)

(a) Introduction to graphs: Types of graphs: Simple graph, Multigraph, psuedograph, directed graph, directed multigraph. One example/graph model of each type to be discussed.

(b) (i) Graph Terminology: Adjacent vertices, degree of a vertex, isoloated vertex, pendant vertex in a undirected graph.
   (ii) The handshaking Theorem for an undirected graph. An undirected graph has an even number odd vertices.

(c) Some special simple graphs: Complete graph, cycle, wheel in a graph, Bipartite graph, regular graph.

(d) Representing graphs and graph isomorphism.

   (i) Adjacency matrix of a simple graph.
   (ii) Incidence matrix of an undirected graph.
   (iii) Isomorphism of simple graphs.

(e) Connectivity:

   (i) Paths, circuit (or cycle) in a graph.
   (ii) Connected graphs, connected components in an undirected graph, A strongly connected directed graph, A weakly connected directed graph. A cut vertex.
   (iii) Connecting paths between vertices.
   (iv) Paths and isomorphisms.
   (v) Euler paths and circuits, Hamilton paths and circuits.
   Dirac's Theorem, Ore's Theorem
   (vi) Shortest path problem, The shortest path algorithm - Dijkstra's Algorithm.

(f) Planar graphs, planar representation of graphs, Euler's formula. Kuratowski's Theorem (statement only).

Reference for Unit 2: *Chapter* 8, *Sections* 8.1, 8.2, 8.3, 8.4, 8.5, 8.6, 8.7 *of* **Discrete Mathematics and Its Applications**, Kenneth H. Rosen, *McGraw Hill Edition.*

Unit 3. Trees (15 Lectures)

(a) (i) Trees: Definition and Examples.
   (ii) Forests, Rooted trees, subtrees, binary trees.
   (iii) Trees as models.
   (iv) Properties of Trees.

(b) Application of Trees:
   (i) Binary Search Trees, Locating and adding items to a Binary Search Tree.
   (ii) Decision Trees (simple examples).
   (iii) Game Trees, Minimax strategy and the value of a vertex in a Game Tree. Examples of games such as Nim and Tic-tac-toe.

(c) Tree Traversal, Traversal algorithm including preorder traversal, inorder traversal, postorder traversal using recursion.

(d) (i) Spanning Tree, Depth-First Search and Breadth-First Search.
   (ii) Minimum Spanning Trees, Prim's Algorithm, Kruskal's Algorithm

(**The Proofs of the results in this unit are not required and may be omitted**).

Reference for Unit 3: *Chapter* 9, *Sections* 9.1, 9.2, 9.3, 9.4, 9.5 *of* **Discrete Mathematics and Its Applications**, Kenneth H. Rosen, *McGraw Hill Edition.*

Term II

Unit 4. Application of Integration (15 Lectures)

(a) (i) Area between two curves.
   (ii) Volumes by slicing, volumes of solids of revolution.
   (iii) Lengths of plane curves.
   (iv) Areas of surfaces of revolution.

(b) (i) Improper integrals of two types:
      (1) The limits of integration (one or two) are infinite,

    (2) Integrand being infinite at one of the end points or an interior point.

  (ii) Convergence of improper integrals, Tests of convergece and divergence.

  Direct comparison test and limit form of comparison test, Evaluation of convergent improper integrals.

  (iii) Applications

    (1) Finding area of an infinite region.
    (2) Volume of solids of revolution of infinite region about x-axis or y-axis.

(c) Euler's Gamma function and Stirling formula.

Reference for Unit 4: *Chapter* 5, *Sections* 5.1, 5.2, 5.3, 5.4, 5.5, 5.6 and *Chapter* 7, *Section* 7.6 *of* **Calculus and Analytic Geometry**, G.B. THOMAS *and* R. L. FINNEY, *Ninth Edition*, *Addison-Wesley*, 1998.

Unit 5. Numerical Methods (15 Lectures)

(a) Roots of equations in one variable: Bisection method, Newton Raphson method, Secant method, Fixed point iteration method and to use it to find roots of equations.

  Convergence, limitations and algorithm for each of the above methods.

(b) Newton Raphson method for a system of non-linear equations, Multiple roots by Newton Raphson method and polynomial deflation.

(c) Roots of polynomial, Fundamental Theorem of Algebra (statement only), Descarte's rule of sign, Muller's method.

(d) *LU* factorization of a matrix where L is a lower triangular matrix and U is an upper triangular matrix, Doolittle LU decomposition, Cholesky decomposition.

Reference for Unit 5: *Chapter* 6, *Sections* 6.1, 6.2, 6.3, 6.5, 6.6, 6.8, 6.9, 6.10, 6.12, 6.13, 6.14, 6.16 and *Chapter 7*, *Section* 7.7 *of* **Numerical Methods**, E. BALAGURUSWAMY, *TATA McGraw Hill*.

Unit 6. Numerical Solution for Ordinary Differential Equations (15 Lectures)

(a) Solution of Initial value problem of an ordinary first order differential equation:

  (i) One step methods: Taylor series method, Picard's method, Euler's method, Heun's method, Polygon method, Runge-kutta method of 2$^{\text{nd}}$ order, 4$^{\text{th}}$ order.
  (ii) Accuracy of one-step methods

(b) Solution of Initial value problem of an ordinary first order differential equation:

(i) Multistep methods (Predictor - Corrector methods): Milne-Simpson method, Adams-Bashforth-Moulton method.

(ii) Accuracy of multistep methods

Reference for Unit 6: *Chapter* 13, *Sections* 13.1, 13.2, 13.3, 13.4, 13.5, 13.6, 13.7, 13.8, 13.9 and *of* **Numerical Methods**, E. Balaguruswamy, *TATA McGraw Hill.*

Recommended Books

1. Kenneth H. Rosen : **Discrete Mathematics and Its Applications**, *McGraw Hill Edition.*
2. Bernard Kolman, Robert Busby, Sharon Ross: **Discrete Mathematical Structures**, Prentice-Hall India.
3. Norman Biggs: **Discrete Mathematics**, *Oxford.*
4. Douglas B. West: **Introduction to graph Theory**, *Pearson.*
5. G.B. Thomas *and* R. L. Finney: *Calculus and Analytic Geometry*, *Ninth Edition*, *Addison-Wesley*, 1998.
6. E. Balaguruswamy: **Numerical Methods**, *TATA McGraw Hill.*

Additional Reference Books

1. Frank Harary, *Graph Theory*, Narosa Publication.
2. R.G. Dromey, *How to Solve it by computers*, Prentice-Hall India.
3. Graham, Knuth and Patashnik: *Concrete Mathematics*, Pearson Education Asia Low Price Edition.
4. Kendall Atkinson : *An Introduction to Numerical Analysis*, Wiley Student Edition.
5. Richard Burden and Douglas Faires: *Numerical Analysis*, Thomson Books/Cole.
6. Thomas H. Cormen, Charles E. Leisenon and Ronald L. Rivest: *Introduction to Algorithms*, Prentice Hall of India, New Delhi, 1998 Edition.

Suggested topics for Practicals

(1) Linear and binary search, sorting.

(2) Algorithms on integers and matrices.

(3) Recursive algorithms.

(4) (i) Drawing a graph, counting the degree of vertices and number of edges.
   (ii) Representing a given graph by an adjacency matrix and drawing a graph having given matrix as adjacency matrix.

(5) Determining whether the given pairs of graphs are isomorphic.

   (Exhibiting an isomorphism between the isomorphic graphs or proving that none exists).

(6) Determining whether the given graph is connected or not.

   Finding connected components of a graph.

   Finding strongly connected components of a graph.

   Finding cut vertices.

(7) To determine whether the given graph is a tree. Construction of Binary Search Tree and applications to sorting and searching.

(8) Spanning Trees. Finding Spanning Tree using Breadth First Search and/or Depth First Search.

(9) (a) Applications of integrals: Finding
      (i) Area between two curves.
      (ii) Volumes by slicing, volumes of solids of revolution.
      (iii) Lengths of plane curves.
      (iv) Areas of surfaces of revolution.
   (b) Improper Integrals.

(10) Bisection Method, Newton Raphson method.

(11) Secant method, fixed point iterative method to find root of equation, Muller's method.

(12) LU decomposition (Doolittle method, Cholesky method).

(13) Euler's method, Heun's method

(14) Runge-Kutta method of 2^{nd} order, 4^{th} order.

(15) Milne-Simpson method, Adams-Bashforth- Moulton method.

Evaluation Scheme and Paper Pattern

Evaluation Scheme (For Paper I and Paper II)

Examination	Maximum Marks	Maximum Marks after conversion
First Term	60	40
Second Term	60	40
Tutorial		10
Assignment/Project		10
Total		**100**

Evaluation Scheme (For Paper III)

Examination	Maximum Marks	Maximum Marks after conversion
First Term	60	30
Second Term	60	30
Practical Examination		40 (35 for experiment 5 for Journal)
Total		**100**

Standard of passing in the subject of Mathematics at S.Y. B. A.

To pass in the subject of Mathematics at S Y B.A, student must secure

at least 4 marks out of 20 in tutorials and assignments/project/case study and

16 marks out of 80 in theory in each paper and

70 marks out of 200 in both the papers taken together.

There will be only one head of passing.

<table>
<tr><td>Paper</td><td colspan="2">Marks in tutorials and assignments / project / case study</td><td colspan="2">Theory</td><td colspan="2">Head of Passing</td></tr>
<tr><td></td><td>Maximum</td><td>Minimum</td><td>Maximum</td><td>Minimum</td><td>Maximum</td><td>Minimum</td></tr>
<tr><td>I</td><td>20</td><td>04</td><td>80</td><td>16</td><td rowspan="2"></td><td rowspan="2"></td></tr>
<tr><td>II</td><td>20</td><td>04</td><td>80</td><td>16</td></tr>
<tr><td>I and II</td><td colspan="4">Theory , Tutorials and Assignments / Project / Case study taken together</td><td>200</td><td>70</td></tr>
</table>

Standard of passing in the subject of Mathematics at S.Y.B.Sc.

To pass in the subject of Mathematics at S Y B. Sc student must secure

(a) at least 4 marks out of 20 in tutorials and assignments/ project/case study and
16 marks out of 80 in theory in each of the Paper I and Paper II and
12 marks out of 60 in Paper III and
91 marks out of 260 in paper I, II and III theory taken together.

(b) At least 14 marks out of 40 in practical of Paper III.

There will be two heads of passing.

<table>
<tr><td>Paper</td><td colspan="2">Marks in tutorials and assignments / project / case study</td><td colspan="2">Theory</td><td colspan="2">Head of Passing</td></tr>
<tr><td></td><td>Maximum</td><td>Minimum</td><td>Maximum</td><td>Minimum</td><td>Maximum</td><td>Minimum</td></tr>
<tr><td>I</td><td>20</td><td>04</td><td>80</td><td>16</td><td rowspan="3"></td><td rowspan="3"></td></tr>
<tr><td>II</td><td>20</td><td>04</td><td>80</td><td>16</td></tr>
<tr><td>III</td><td colspan="2"></td><td>60</td><td>12</td></tr>
<tr><td>I and II
III</td><td colspan="4">Theory, Tutorials and Assignments / Project / Case study taken together</td><td>260</td><td>91</td></tr>
<tr><td>III</td><td colspan="4">Practical</td><td>40</td><td>14</td></tr>
</table>

- If a student fails to get minimum 4 marks in tutorials and assignments / project / case study in any of the paper, the student shall submit tutorial and assignments / project / case study within two months after declaration of the result. The appropriate ordinances and regulations will be applicable after evaluating tutorials and assignments / project/ case study. Failure of submission within the stipulated time will result in his/her not passing.
- Marks in Tutorial and Assignments / Project / Case study taken together (out of 20) in each paper shall be shown separately on statement of marks.
- If a student fails in theory (term end examination) and has secured at least 4 marks in tutorials and assignments / project / case study in each of the paper, the marks in tutorials and assignments / project / case study will be carried further. The appropriate ordinances and regulations will be applicable after evaluating for term end examination.

Paper Pattern - Term End Examination

For Paper I, Paper II and Paper III

Duration for Term End Examination - 2 Hrs.
Maximum marks - 60.
All questions are compulsory.

Questions	Term I	Term II	Maximum Marks	Maximum Marks with Options
Q1	Based on Unit 1, Unit 2 and Unit 3	Based on Unit 4, Unit 5 and Unit 6	15	22 or 23
Q2	Based on Unit 1	Based on Unit 4	15	22 or 23
Q3	Based on Unit 2	Based on Unit 5	15	22 or 23
Q4	Based on Unit 3	Based on Unit 6	15	22 or 23
	Total Marks		**60**	

Paper Pattern - A.T.K.T

For Paper I, II and III

Duration for A.T.K.T Examination - 3 Hrs.
All questions are compulsory.

Questions	Section I	Maximum Marks	Maximum Marks with Options
Q1	Based on Unit 1	15	22 or 23
Q2	Based on Unit 2	15	22 or 23
Q3	Based on Unit 3	15	22 or 23
	Section II		
Q4	Based on Unit 4	15	22 or 23
Q5	Based on Unit 5	15	22 or 23
Q6	Based on Unit 6	15	22 or 23
	Total Marks	**90**	

In A.T.K.T. Examination, the marks secured by the student in each of the Paper I and Paper II, out of 90 to be converted out of 80, rounded off to the next integer. In Paper III, the marks out of 90 to be converted out of 60, rounded off to the next integer.

Guidelines about conduct of Tutorials/Assignments/Projects/Case Study.

1. **Tutorials**

   Conduct and Evaluation: The tutorials should be conducted in batches formed as per the University guidelines. During each tutorial class, there should be a discussion of the tutorial question between the teacher and the student. The same should be entered in the tutorial book. Each tutorial should be evaluated out of 10 marks on the basis of participation and performance of a student and the average of the total aggregate should be taken. The tutorial note book should be maintained throughout the year and should be certified by the concerned teacher and the head of the department/senior teacher in the department.

2. **Assignments**

   Conduct and Evaluation: The topic of the assignment and the questions should be given to the students at least one week in advance. The duration of assignment should be 45 - 50 minutes. The teachers may resolve the doubts of the students during the week, after which the students should submit the assignment. Each assignment should be evaluated out of 10 marks and the average of the total aggregate should be taken. The assignment note book should be maintained throughout the year and should be certified by the concerned teacher and the head of the department/senior teacher in the department.

3. **Projects/Case Study**

   Conduct and Evaluation: A student may submit a project/case study in paper I and/or Paper II instead of assignment in that paper. The student should submit a project/case study report in Mathematics of about 10 typed pages (the number of pages should not exceed 20). The topic of the project/case study should be selected in consultation with the teacher.

   The topic can be of expository / historical survey / interdisciplinary nature and the material covered in the project/case study should go beyond the scope of the syllabus. The student should clearly mention the sources (book/on-line) used for the project/case study. The use of Computer Algebra System (CAS) such as Mathematical softwares should be encouraged. The project/case study may be done under the supervision of a faculty member in a College / Institution / University of Mumbai.

   The following marking scheme is suggested for the evaluation of projects:

   30 percent marks: exposition

   20 percent marks: literature

   20 percent marks: scope

   10 percent marks: originality

   20 percent marks: oral presentation